A Historical Perspective on the Use of Manipulatives

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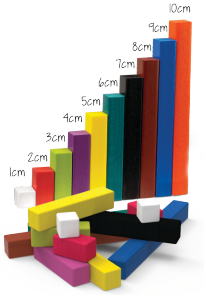
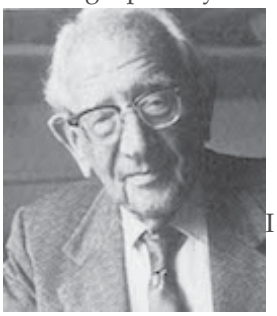


George Pòlya (1887–1985) was an international scholar, an excellent mathematician, and a professor of mathematics at Stanford University for decades. He was also a co-founder of the California Mathematics Council (CMC). His workshops, presentations, and writings for teachers have revolutionized mathematics instruction. Generations of learners of mathematics have fallen in love with mathematics by participating in problem-solving activities and in mathematics competitions that he inspired. George Pòlya stated the following:

> The two principles of active learning—priority of action and perception—are taken into account by almost all mathematics teaching today. However, there is allegedly a Chinese proverb that says, "I hear and I forget. I see and I remember. I do and understand." So "I hear and I forget." What you just hear you forget quickly. Good advice is very quickly forgotten. What you see with your own eyes is remembered better, but you really understand it when you do it with your own hands. So there must be more than just priority of action and perception in our teaching. (CMC Infinity Wall, **https://www.cmc-math.org/george-polya**).

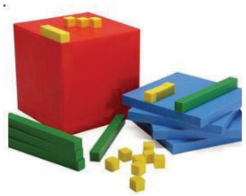


The use of various manipulatives as tools for conveying mathematical concepts and for solving problems is a natural part of communication and thinking for many people. However, most of the manipulatives are ad-hoc inventions and not a common part of the "toolboxes" of teachers around the world. A great legacy of the mathematics educational reforms of the 1960s provided workshops that helped teachers learn ways of effectively using classroom sets of redesigned and organized manipulatives. The most widely analyzed and promoted manipulatives of the 1960s were Cuisenaire rods, geoboards, and multi-base arithmetic blocks (also known as "Dienes blocks").

Georges Cuisenaire (1891–1975) was a Belgian elementary school teacher who invented Cuisenaire rods as a mathematics teaching aid. In the 1950s, Caleb Gattegno (1911–1988), who taught mathematics at all levels and was very active in mathematics education internationally, became the main advocate of Cuisenaire rods, and is considered to be the designer of geoboards. Gattegno invented and promoted innovative approaches to teaching and learning mathematics (*Visible & Tangible Math*), foreign languages (*The Silent Way*), and reading (*Words in Color*). He was a prolific writer, authoring more than 120 books and hundreds of articles, mostly on the topics of education and human development. Teachers used Cuisenaire rods and geoboards mostly in the teaching of young children. However, Pòlya also used them in instructing graduate-level mathematics classes because he found them so valuable (Taylor & Taylor, 1993, p. 75). Cuisenaire rods are very useful in exploring concepts such as partitions of quantities and equivalent fractions, as well as greatest common factors and least common multiples of numbers. Teachers use geoboards to find the area and perimeter of polygons, but they can also use them for exploring theorems, such as Pick's theorem.

One of the early promoters of familiarizing young students with wooden geometrical objects was the German educator Friedrich Froebel (1782–1852). He also was the inventor of the word and the institute of "Kindergarten" as the "Play and Activity Institute for Small Children." Children played, sang, danced, and used any other means of active learning as a preparation for their lives. Caroline Pratt

(1867–1954) popularized unit blocks in the early 1900s. The Montessori education system (Montessori, 1965) has illustrated division with rulers, squares, and cubes since 1914 (Servais & Varga, 1971, p. 105). This system was developed by Maria Montessori (1870–1952), who became the first Italian female physician after graduating with highest honors from medical school. For details about her fascinating life and work, see **https://education.stateuniversity.com/pages/2244/Montessori-Maria-1870-1952.html.**



The unique contributions of Zoltan Paul Dienes (1916–2014) related to the use of arithmetic blocks. These manipulatives systematically use blocks with various bases (hence the name "*Multi-base* arithmetic blocks"), highlighting the structure of place-value systems for recording numbers and doing operations with them, depending on the base number chosen. By using these manipulatives, the observer can conduct experimental studies on the ways students learn, struggle with, and master the use of those structures with observable and recordable actions by the learner. The results can be replicated by conducting the same kinds of activities in classrooms all around the world through the use of these manipulatives (Tuska, 2018). A good example of the experimental studies related to multi-base arithmetic blocks is the book *Mathematics Learning Project* by Jerome Bruner and Dienes (see Dienes, 1963). Patricia Moyer provided a valuable summary of the research background for the use of manipulatives (Moyer, 2001, pp. 175–176), referring to the works of Piaget, Bruner, Skemp, Vygotsky, Cobb, and others, in addition to Dienes.

The CMC's "Infinity Wall" also highlights Mary Laycock's work in mathematics education. She gave the following powerful testimonial to the work of Dienes:

> I had all of the Dienes' books and the Dienes' blocks. As I tried them with children, I saw their excitement and growth in understanding. As I read Dienes' material, I saw what he meant as I learned to watch children's fingers to understand what went on in their heads. Dienes' material is outstanding if the teacher has a mathematical background. My gift has been to make Dienes' message understandable to teachers. (CMC Infinity Wall, **https://www.cmc-math.org/mary-laycock**).

The original multi-base arithmetic blocks have mostly been replaced by interlocking cubes that allow building models for understanding arithmetic in any base system, and by algebra tiles for generalizing the concepts of arithmetic to a "base x system." Both of these manipulatives are presently in the cabinets of many teachers. Teachers should not forget to read the books of Dienes. They are timeless gems!

Currently, technology opens up many new learning opportunities and virtual manipulatives and explorations have become widely used. For example, see the National Library of Virtual Manipulatives at **http://nlvm.usu.edu/en/nav/vlibrary.html**, or explorations offered by Desmos at **https://www.desmos.com**. The dynamic views offered by educational films of Nicolet in the 1940s (Servais & Varga, 1971, pp. 101–102) are still fascinating to watch. See **https://www.youtube.com/watch?v=gum9kvxR9K8**. The films can be enhanced or replaced by hands-on experiences with dynamic software, such as SketchUp **(https://www.sketchup.com/)**, or GeoGebra **(https://www.geogebra.org/).** GeoGebra is my favorite for being a versatile open-source software that a large international community of volunteers continuously perfect. Also, it showcases an excellent collection of classroom resources. See the free activities, simulations, exercises, lessons, and games at **https://www.geogebra.org/materials.**

Yet, these technological tools should never replace the use of real-touch objects and physical manipulatives in the learning process. The use of touch, proprioception (the perception of our bodies), and kinesthesia (self-initiated body motion) are irreplaceable in the effective, engaged, and active brain during the process of learning mathematics.

*See **Historical** (continued page 25)* ☞☞